ISSN-e: 1856-9811

ON QUADRIPARTITIONED SINGLE-VALUED NEUTROSOPHIC POLYGROUPS

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Abstract

Studying neutroalgebra is becoming one of the most important topic of great interest for many researchers around the world, where neutron-subgroup, neutro BL-algebra, antialgebra, quadripartitioned single-valued neutro algebra, single-valued neutrosophic polygroup and others have been defined. This is why, the objective of this work is to use the notions of quadripartitioned single-valued neutrosophic set, polygroup and single-valued neutrosophic set to define the concept of quadripartitioned single-valued neutrosophic polygroup. Furthermore, we show some of its properties and prove the relation between level sets of quadripartitioned single valued neutrosophic polygroups and subpolygroups

Keywords: *Neutrosophic logic, quadripartitioned single-valued, Polygroup, quadripartitioned single-valued polygroup, quadripartitioned anti-single-valued polygroup.*

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RECEIVED: 09-07-2025 / ACCEPTED: 13-09-2025 / PUBLISHED: 22-12-2025

How to quote: Granados et al. (2025). On quadripartitioned single-valued neutrosophic polygroups. *Anales*, 42, 1 - 16. https://doi.org/10.58479/anales.2025.140

ISSN-e: 1856-9811

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1. Introduction:

Neutrosophy is a new branch of philosophy which was introduced by (Smarandache, 2002) which has been of great interesting of researchers who study different topics (applied or pure science) and it studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra: (B) is an idea, proposition, theory, event, concept or entity; anti (B) is the opposite of (B); and (neut-B) means neither (B) nor anti (B), i.e. neutrality between the two extremes (Bal eta l., 2018).

Many papers have been done for many researchers related to neutroalgebra. A neutroalgebra is an algebra which has at least one neutrooperation or one neutroaxiom (axiom that is true for some elements, indeterminate for other elements, and false for the other elements) (Smarandache, 2020).

With a view to device a practical tool for inference, Belnap (1977) introduced the notion of a four valued logic. In his work, corresponding to a certain information he considered four possibilities namely T: True, F: False, none: neither true nor false and both: both true and false. He symbolized these four truth values as {T, F, both, none}, for more notions derived from this paper, we refer the reader to (Das, et al., 2021; Mohanasundari and Mohana, 2020).

In this work, we carry out the notion of quadripartitioned single-valued neutrosophic polygroup which is an extension of the paper presented by (Al-Tahan, 2020). Besides, we show some properties on quadripartitioned single-valued neutrosophic polygroup.

2. Preliminaries:

In this section, we recall some well-known notions which are of our interested for the development of this work.

Definition 2.1. Let Y be a space of points (objects), with a generic element in Y denoted by y. A quadripartitioned neutrosophic set B in Y is characterized by a truth-membership function T_B, a contradiction membership function $C_{B,}$ an ignorance membership function I_B and a falsity membership function F_B. $T_B(y)$, $C_B(y)_{,}$ $I_B(y)$ and $F_B(y)$ are real standard or non-standard subsets of

]0−, 1+[. There is no restriction on the sum of $T_B(y)$, $C_B(y)$, $I_B(y)$ and $F_B(y)$, thus $0^- \leq sup\ T_B(y) + sup\ C_B(y)$

$+ sup\ I_B(y) + sup\ F_B(y) \leq 4^+$ (Chatterjee et al., 2016).

Example 2.2. Consider that Y= {w,,y,z}, w is knowledge, x is wrong knowledge y is more and less knowledge and z is bad knowledge. The values of w,x,y, and z are in [0,1]. They are obtained from the questionnaire about general culture, their option could be a degree of “good knowledge”, a degree of contradiction, a degree of ignorance or a degree of “poor knowledge”. B is a quadripartitioned single valued neutrosophic set of Y defined by

B=<0.3,0.1,0.5,0.7>/w +<0.4,0.2,0.1,0.6>/x+<0.5,0.2,0.8,0.1>/y+ <0.7, 0.9,0.1,0.3>/z

And D is a quadripartitioned single-valued neutrosophic set of Y defined by D=<0.4,0.3,0.1,0.8>/w +<0.5,0.3,0.6,0.1>/x+<0.9,0.1,0.5,0.3>/y+ <0.3, 0.1,0.8,0.5>/z

Definition 2.3. Let Y be a space of points (objects),with a generic element of Y denoted by y. A quadripartitioned single-valued neutrosophic set (QSVNS) B in Y is characterized by T_B, C_B, I_B and F_B. For each point y in Y, and T_B, C_B, I_B , F_B ∈[0,1] (Chatterjee et al., 2016).

Definition 2.4. The complement of a quadripartitioned single-valued neutrosophic set B is denoted by C(B) and is defined by

I. $T_{C(B)}(y)=F_B(y)$,

II. $C_{C(B)}(y)=1-CB(y)$,

III. $I_{C(B)}(y)=1-IB(y)$,

IV. $F_{C(B)}(y)=T_B(y)$.

For all y in Y (Chatterjee et al., 2016).

Example 2.5. Let B be the quadripartitioned single-valued neutrosophic set present in Example 2.2. Then

C(B) = B=<0.7,0.9,0.5,0.3>/w +<0.6,0.8,0.9,0.4>/x+<0.1,0.8,0.2,0.5>/y+ <0.3, 0.1,0.9,0.7>/z

Definition 2.6. A quadripartitioned single-valued neutrosophic set B is contained in the other quadripartitioned single-valued neutrosophic set D, B⊆D, if

I. $T_B(y) \leq TD(y)$

II. $C_B(y) \leq CD(y)$

III. $I_B(y) \geq ID(y)$

IV. $F_B(y) \geq FD(y)$

For all y in Y (Chatterjee et al., 2016).

Definition 2.7. The union of two quadripartitioned single-valued neutrosophic sets B and D is a quadripartitioned single-valued neutrosophic set E, written as E=B∪D, whose truth, contradiction, ignorance and falsity-membership functions are given by,

I. $T_E(y)=\max(T_B(y),T_D(y))$,

II. $C_E(y)=\max(C_B(y),C_D(y))$,

III. $I_E(y)=\min(I_B(y),I_D(y))$,

IV. $F_E(y)=\min(F_B(y),F_D(y))$.

For all y in Y (Chatterjee et al., 2016).

Definition 2.8. The intersection of two quadripartitioned single-valued neutrosophic sets B and D is a quadripartitioned single-valued neutrosophic set E, written as E=B∩D, whose truth, contradiction, ignorance and falsity-membership functions are given by,

I. $T_E(y)=\min(T_B(y),T_D(y))$,

II. $C_E(y)=\min(C_B(y),C_D(y))$,

III. $I_E(y)=\max(I_B(y),I_D(y))$,

IV. $F_E(y)=\max(F_B(y),F_D(y))$.

For all Y in y (Chatterjee et al., 2016).

Example 2.9. Let B and D be the quadripartitioned single-valued neutrosophic sets present in

Example 2.2. Then

B ∩ D = <0.3,0.1,0.5,0.8>/w + <0.4,0.2,0.6,0.6>/x + <0.5,0.1,0.8,0.3>/y+ >0.3,0.1,0.8,0.5>/z

And

B ∪ D = <0.4,0.3,0.1,0.7>/w + <0.5,0.3,0.1,01>/x + <0.9,0.2,0.5,0.1>/y+ >0.7,0.9,0.1,0.3>/z

Next, we show the notion of polygroup and some of its basics properties (Comer, 1984; Davvaz et al., 2015, Davvaz et al., 2013).

Definition 2.10. Let M be a non-empty set and $P^{\circ}(M)$ be the collection of all non-empty subsets of M. "°" is defined as follows:

$$\circ\ M\times M \rightarrow P^{\circ}(M)$$

$$(q,w)\rightarrow q^{\circ}w$$

Then, "°" is said to be a hyperoperation and (M,°) is called a hypergrupoid.

Definition 2.11. Let (M,°) be a hypergrupoid. Then, (M,°) is a polygroup if the following statements are satisfied for all x,y,z in M.

I. $x^{\circ}(y^{\circ}z)=(x^{\circ}y)^{\circ}z$,

II. There exists e in M with $e^{\circ}x=x^{\circ}e=x$, for all x in M,

III. $x\in y^{\circ}z$ implies $y \in x^{\circ}\boldsymbol{z^{-1}}$ and $z\in \boldsymbol{y^{-1}}{}^{\circ}\ x$.

Remark 2.12. Weak polygroups are generalization of polygroups and they are defined in the same way as polygroups but instead of (I) in Definition 2.11, we have $x^{\circ}(y^{\circ}z) \cap (x^{\circ}y)^{\circ}z \neq \emptyset$. Besides, in weak polygroup M, $(\boldsymbol{x^{-1}})^{-1} = \mathbf{1}$ for all x in M.

Remark 2.13. Al-Tahan (2020) mentioned that every group is a weak polygroup, but converse is not always true. (See Example 3.1 in (Al-Tahan, 2020)).

Example 2.14. Let M={e,b,c,d}. Then $(M_3,^{\circ})$ defined in Table 1, it is a weak polygroup with e serving as an identity. Furthermore, it is not a polygroup.

$\circ$	e	b	c	d
e	e	b	c	d
b	b	{e,b}	d	c
c	c	d	{e,c}	b
d	d	c	b	{e,d}

Table 1. Weak poligroup (M_3,°) (Davvaz,2013).

Definition 2.15. Let (M,°) be a polygroup. A subset R of M is subpolygroup of M if (R,°) is a polygroup.

Proposition 2.16. Let (M,°) be a polygroup. A subset R of M is subpolygroup of M if a°b ⊆ R and

$\boldsymbol{a^{-1}}$ ∈ R for all a, b ∈ R.

Definition 2.17. Let $(P,°)$ be a polygroup. A subset subpolygroup R of M is a normal subpolygroup of M if $\boldsymbol{a^{-1}}$°M°a ⊆ M for all a ∈ M.

3. Quadripartitioned single-valued neutrosophic polygroup

In this section, we introduce the notion of quadripartitioned single-valued neutrosophic polygroups and study some of its properties. Additionally, we define level sets of quadripartitioned single valued neutrosophic polygroups and relate them to normal subpolygroups.

Definition 3.1. Let (M,°) be a weak polygroup and B be a quadripartitioned single-valued neutrosophic set over M. Then, B is said to be a quadripartitioned single-valued neutrosophic polygroup (QSVNP) over M (quadripartitioned single-valued neutrosophic weak polygroup (QSVNWP) over M) if for all a,b in M, the following statements hold:

V. T_B(d) ≥ min{ TB(a), T_B(b)}, C_B(d) ≥ min{ CB(a), C_B(b)}, I_B(d) ≤ max{ IB(a), I_B(b)} and F_B(d)

≤ max{ FB(a), F_B(b)} for all c in a°b.

VI. $T_B(\boldsymbol{a^{-1}}) \geq TB(a)$, $C_B(\boldsymbol{a^{-1}}) \geq CB(a)$, $I_B(\boldsymbol{a^{-1}}) \leq IB(a)$ and $F_B(\boldsymbol{a^{-1}}) \leq FB(a)$.

Example 3.2. Let $(M_1,°)$ be a polygroup defines below:

°	0	1
0	0	1
1	1	M_1={0,1}

Where 0 serves as an identity. Then, B=<0.3,0.5,0.5,0.7>/0 +<0.4,0.2,0.4,0.6>/1 is a quadripartitioned single-valued neutrosophic polygroup over M_1.

Remark 3.3. All the theorems and results present in this work that are valid for quadripartitioned single-valued neutrosophic polygroup are also valid for quadripartitioned single-valued neutrosophic weak polygroup. Thus, we restrict our results to quadripartitioned single-valued neutrosophic polygroup.

Proposition 3.4. Let (M,°) be a polygroup and B be a quadripartitioned single-valued neutrosophic polygroup over M. Then, the following statements hold for all y∈ M:

I. $T_B(\boldsymbol{y^{-1}})= T_B(y)$, $C_B(\boldsymbol{y^{-1}})= C_B(y)$, $I_B(\boldsymbol{y^{-1}})= I_B(y)$ and $F_B(\boldsymbol{y^{-1}})= F_B(y)$,

II. $T_B(e) \geq TB(y)$, $C_B(e) \geq CB(y)$, $I_B(e) \leq IB(y)$ and $F_B(e) \leq FB(y)$ where e is the identity in M.

Proof. Let y∈M, then

I. By Definition 3.1, $T_B(\boldsymbol{y^{-1}}) \geq TB(y)$, $C_B(\boldsymbol{y^{-1}}) \geq CB(y)$, $I_B(\boldsymbol{y^{-1}}) \leq IB(y)$ and $F_B(\boldsymbol{y^{-1}}) \leq FB(y)$, and taking $(\boldsymbol{y^{-1}})^{-1} = \boldsymbol{y}$ implies that $T_B(\boldsymbol{y^{-1}}) \leq TB(y)$, $C_B(\boldsymbol{y^{-1}}) \leq CB(y)$, $I_B(\boldsymbol{y^{-1}}) \geq IB(y)$ and $F_B(\boldsymbol{y^{-1}}) \geq FB(y)$. Therefore, $T_B(\boldsymbol{y^{-1}})= T_B(y)$, $C_B(\boldsymbol{y^{-1}})= C_B(y)$, $I_B(\boldsymbol{y^{-1}})= I_B(y)$ and $F_B(\boldsymbol{y^{-1}})= F_B(y)$.

II. Since $e\in y°\boldsymbol{y^{-1}}$, it follows by Definition 3.1 part (I) that $T_B(e) \geq \min\{TB(y),T_B(\boldsymbol{y^{-1}})\}=T_B(y)$, $C_B(e)$

$\geq \min\{CB(y),C_B(\boldsymbol{y^{-1}})\}=C_B(y)$, $I_B(e) \leq \max\{IB(y),I_B(\boldsymbol{y^{-1}})\}=I_B(y)$ and $F_B(e)$

$\leq \max\{FB(y),F_B(\boldsymbol{y^{-1}})\}=F_B(y)$.

Example 3.5. Let (M_2, °) be the polygroup defined below.

°	e	q	w	r
e	e	q	w	r
q	q	e	w	r
w	w	w	{e,q,r}	{w,r}
r	r	r	{w,r}	{e,q,w}

Then b = <0.1,0.5,0.5,0.8>/e + <0.6,0.7,0.2,0.4>/x +<0.4,0.7,0.8,0.1>/y+<0.1,0.5,0.6,0.9>/z is not a

quadripartitioned single-valued neutrosophic polygroup over M_2 as $T_B(e)$ ≥TB(x) does not hold.

Proposition 3.6. Let (M,°) be a polygroup, B be a quadripartitioned single-valued neutrosophic set over M, and $\boldsymbol{B^{-1}}$ = { <$T_B(\boldsymbol{y^{-1}})$, $C_B(\boldsymbol{y^{-1}})$, $I_B(\boldsymbol{y^{-1}})$, $F_B(\boldsymbol{y^{-1}})$>: y∈ M}. If B is a quadripartitioned

single-valued neutrosophic polygroup over. Then, $\boldsymbol{B^{-1}} = \boldsymbol{B}$.

Proof. The proof follows from Proposition 3.4.

Proposition 3.7. Let (M,°) be a polygroup and t_0,t_1,t_2,t_3 be numbers in the unit interval [0, 1]. If B=

{<t_0,t_1,t_2,t_3>/y: y ∈ PM}. Then, B is a quadripartitioned single-valued neutrosophic polygroup over M.

Proof. The proof follows from Propositions 3.4 and 3.6.

Remark 3.7. The quadripartitioned single-valued neutrosophic polygroup present in Proposition 3.6 is said to be the constant quadripartitioned single-valued neutrosophic polygroup.

Theorem 3.8. Let (M,°) be a polygroup and B be a quadripartitioned single-valued neutrosophic set over M. Then, B and C(B) are quadripartitioned single-valued neutrosophic sets over M if and only if B is the constant quadripartitioned single-valued neutrosophic polygroup.

Proof. Let B and C(B) be quadripartitioned single-valued neutrosophic polygroup. Then, for all y in M, we get

I. TB(e) ≥ TB(y), $C_B(e) \geq C_B(y)$, $I_B(e) \leq I_B(y)$ and FB(e) ≤ FB(y),

II. FB(e) ≥ F(y), $1 - C_B(e) \geq 1 - C_B(y)$, $1\text{-}I_B(e) \leq 1\text{-}I_B(y)$ and TB(e) ≤ TB(y).

These imply that $T_B(e)=T_B(y)$, $C_B(e)=C_B(y)$, $I_B(e)=I_B(y)$ and $F_B(e)=F_B(y)$. Therefore, B is the constant quadripartitioned single-valued neutrosophic polygroup. If B is the constant quadripartitioned single-valued neutrosophic polygroup over M, then C(B) is also the constant quadripartitioned single valued neutrosophic polygroup over M.

Definition 3.9. Let (M,°) be a polygroup and B be a quadripartitioned single-valued neutrosophic set over M. Then B is called a quadripartitioned anti-single-valued neutrosophic polygroup (QASVNP) over M if for all q, w ∈ M, the following conditions are satisfied.

I. TB(r) ≤ max{TB(q), TB(w)}, $C_B(r) \leq \max\{C_B(q), C_B(w)\}$, $I_B(r) \geq min\{I_B(q), I_B(w)\}$ and FB(r) ≥

min{FB(q), FB(w)} for all r ∈q∗w,

II. TB($\boldsymbol{q}^{-1}$) ≤ TB(q), $C_B(\boldsymbol{q}) \leq C_B(q)$, $I_B(\boldsymbol{q}) \geq I_B(q)$ and FB($\boldsymbol{q}$) ≥ FB(q).

Theorem 3.10. Let (M,°) be a polygroup and B be a quadripartitioned single-valued neutrosophic set over M. Then, B is a quadripartitioned single-valued neutrosophic polygroup over M if and only if C(B) is a quadripartitioned anti-single-valued neutrosophic polygroup over M.

Proof. Let B be a quadripartitioned single-valued neutrosophic polygroup. By Theorem 4.2 of (Al-Tahan, 2020) asserts that T_B, I_B and C_B are fuzzy polygroups over M and F_B is an anti-fuzzy

polygroup over M. So, we have now that $T_{C(B)}=F_B$, $I_{C(B)}=1\text{-}I_B$ and $C_{C(B)}=1\text{-}C_B$ are anti-fuzzy polygroups over M and $F_{C(B)}=T_B$ is a fuzzy polygroup over M. Therefore, this completes the proof. Similarly, it can be proved that C(B) is a quadripartitioned anti-single-valued neutrosophic polygroup over M. Hence, B is a quadripartitioned single-valued neutrosophic polygroup.

Next, we define level sets of quadripartitioned single-valued neutrosophic polygroups and relate them to subpolygroups.

Definition 3.11. Let Y be any set, $t=(t_1, t_2, t_3, t_4)$ where $0 \leq t1, t_2, t_3 < 1$ and $0 < t_4 \leq 1$, and B be a quadripartitioned single-valued neutrosophic set over Y. Then, $B_t=\{y \in Y: T_B(y) \geq t1, C_B \geq t2, I_B \leq t_3, F_B \leq t_4\}$ is called a t-level set of B.

Theorem 3.12. Let B be a quadripartitioned single-valued neutrosophic polygroup over M and a, b ∈

$B_t \neq \emptyset$. For all c ∈ a ° b, we have TB(c) ≥ min {TB(a), TB(b)} ≥ $t1$, C_B(c) ≥ min {C_B(a), C_B(b)} ≥ $t2$, I_B(c) ≤ max

{I_B(a), I_B(b)} ≤t3 and FB(c) ≤ max {FB(a), FB(b)} ≤ $t4$. Thus, a ° b ⊆ B_t. Furthermore, having TB($\boldsymbol{a^{-1}}$) ≥

TB(a) ≥ $t1$, $C_B(\boldsymbol{a^{-1}})$ ≥ C_B(a) ≥ $t2$, $I_B(\boldsymbol{a^{-1}})$ ≤ I_B(a) ≤ $t3$ and $F_B(\boldsymbol{a^{-1}})$ ≤ F_B(a) ≤ $t4$ implies that $\boldsymbol{a^{-1}}$ ∈ B_t. Thus, B_t is a subpolygroup of M.

Conversely, let $B_t \neq \emptyset$ be a subpolygroup of M and a, b ∈ M. Set t_1 = min {TB(a), TB(b)}, t_2 = min {C_B(a), C_B(b)}, t_3 = max {I_B(a),I_B(b)} and t_4 = min {F_B(a), F_B(b)}, and t = ($t1$, $t2$, $t3$, t_4). Since B_t is a subpolygroup of B, it follows that a ° b ⊆ B_t and $\boldsymbol{a^{-1}}$ ∈ B_t. The latter implies that for all c ∈ a ° b, TB(c) ≥ $t1$ = min

{TB(a), TB(b)}, C_B(c) ≥ $t2$ = min {C_B(a),C_B(b)}, I_B(c) ≤ $t3$ = max {I_B(a), I_B(b)} and F_B(c) ≤ $t4$ = max {F_B(a), F_B(b)}. Moreover, we have TB($\boldsymbol{a^{-1}}$) ≥ $t1$ = TB(a), $C_B(\boldsymbol{a^{-1}})$ ≥$t2$ = C_B(a), $I_B(\boldsymbol{a^{-1}})$ ≤$t3$ = I_B(a) and $F_B(\boldsymbol{a^{-1}})$ ≤$t4$ = F_B(a). Therefore, B is a quadripartitioned single-valued neutrosophic polygroup over M.

Example 3.13. Let M_1 = {0, 1} and (M_1,°)be the polygroup defined in Example 3.2. Then the constant quadripartitioned single-valued neutrosophic polygroup and B = ⟨t_1, t_2, t_3, t_4⟩/1 + ⟨t'1, t'2, t'3, t'4⟩/0 where $t1 \leq t'1$, $t2 \leq t'2$, t_3≤t'3 and $t4 \geq t'4$ are the only quadripartitioned single-valued neutrosophic polygroup over M_1.

Theorem 3.14. Let (M,°) be a polygroup. Then every subpolygroup of M is a level set of a quadripartitioned single-valued neutrosophic polygroup over M.

Proof. Let W be a subpolygroup of M and t=(a,b c,d) where 0< a,b,c<1 and 0<d<1. Define the quadripartitioned single-valued neutrosophic set over M as follows:

$$\boldsymbol{B(y)} \begin{cases} (\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}), & \boldsymbol{if\ y \in W}, \\ (\boldsymbol{0}, \boldsymbol{0}, \boldsymbol{0}, \boldsymbol{1}), & \boldsymbol{otherwise}. \end{cases}$$

Let t'=(a',b',c',d'). Then,

$$B' = \begin{cases} \boldsymbol{W}, & \boldsymbol{if\ a > a', b > b', c > c' and\ d < d'} \\ \boldsymbol{M}, & \boldsymbol{if\ a' = 0, b' = 0, c' = 0\ and\ d' = 1, is\ either\ \emptyset\ or\ asubpolygroup\ of\ M} \\ \emptyset, & \boldsymbol{Otherwise}. \end{cases}$$

By Theorem 3.12, we have that B is a quadripartitioned single-valued neutrosophic polygroup over M.

Definition 3.15. Let (M,°) be a polygroup and b be a quadripartitioned single-valued neutrosophic polygroup over M. Then, B is said to be a normal quadripartitioned single-valued neutrosophic polygroup over M if B(w) = A(w') for all w ∈ $x \circ y$, z' ∈ $y \circ x$.

Example 3.16. Let (M,°) be a polygroup and B be a quadripartitioned single-valued neutrosophic polygroup over M. Then, the constant quadripartitioned single-valued neutrosophic polygroup is a normal quadripartitioned single-valued neutrosophic polygroup over M.

Theorem 3.17. Let (M,°) be a polygroup and B is a quadripartitioned single-valued neutrosophic set over M. Then, B is a normal quadripartitioned single-valued polygroup over M, if and only if, $B_t \neq \emptyset$ is a normal subpolygroup of M for every t = (a, b, c, d) where 0 ≤ a, b, c < 1 and 0 < d ≤ 1.

Proof. Let B be a normal quadripartitioned single-valued polygroup over M and q, w ∈ $B_t \neq \emptyset$. By Theorem 3.12, we assert that $B_t \neq \emptyset$ is a subpolygroup of M. Let q ∈ M. We need to show that $\boldsymbol{q^{-1}} \circ B_t$

$\circ$ q ⊆ B_t. Let e ∈ $\boldsymbol{q^{-1}} \circ B_t \circ$ q. Then, there exists w in B_t such that e ∈ $\boldsymbol{q^{-1}} \circ$ w ∘ q and hence e ∈ $\boldsymbol{q^{-1}}$ ∘ s where s ∈ w ∘ q. The latter implies that w ∈ s ∘ $\boldsymbol{q^{-1}}$. And since B is a normal quadripartitioned single-valued polygroup over M, it follows that B(e) = B(w). Thus, e ∈ B_t.

Conversely, let $B_t \neq \emptyset$ be a normal subpolygroup of M. By Theorem 3.12, we assert that B is a quadripartitioned single-valued polygroup over M. To show that B is a normal quadripartitioned single-valued polygroup over M, it suffices to show that B(e) = B(e') for all e ∈ q ∘ w, z' ∈ w ∘ q. Let e

∈ q ∘ w, z' ∈ w ∘ q with B(e′) = t. Having e' ∈ w ∘ q implies that w ∈ z' ∘ $\boldsymbol{q^{-1}}$. The latter implies that Let e ∈ w ∘ e' ∘ $\boldsymbol{q^{-1}}$. Since e' ∈ B_t and $B_t \neq \emptyset$ is a normal subpolygroup of M, it follows that e ∈ B_t and hence, B(e) ≥ B(e') = t. Similarly, we get that B(e') ≥ B(e).

Example 3.18. Let M_2 = {e, q, w, r} and (M_2, ∘) be the polygroup defined in Example 3.5. Then the constant quadripartitioned single-valued neutrosophic polygroup is the only normal quadripartitioned single-valued neutrosophic polygroup over M_2.

Corollary 3.19. Let (M,°) be a polygroup and B be a quadripartitioned single-valued neutrosophic polygroup over M. Then, B° = {y ∈ M: B(Y) = B(e)} is a subpolygroup of M. Moreover, if B is a normal quadripartitioned single-valued neutrosophic polygroup over M, then B° is a normal subpolygroup of M.

Proof. Let t = B(e). Then B_t = {y ∈ M: TB(y) ≥ TB(e), C_B(y) ≥ C_B(e), C_B(y) ≤ C_B(e), FB(y) ≤ FB(e)}. By proposition 4.4 and 3.6, we have that B_t= {y ∈ M: TB(y) = TB(e), IB(y) = IB(e), C_B(y) = C_B(e), FB(y) =

FB(e)} = A°. Therefore, by Theorem 3.12 and Theorem 3.17, we complete the proof.

4. Conclusions

In this work, we have introduced an algebraic hyperstructure of polygroup for the quadripartitioned single-valued neutrosophic set by defining several hyperalgebraic structures and some properties were proved. The notions that were defined were namely quadripartitioned single-valued neutrosophic polygroups and quadripartitioned anti- single-valued neutrosophic polygroups, the results obtained in this work can be considered as a generalization for the work related to single-valued neutrosophic polygroups. For future work associated to this work involve the development of hyperalgebraic for the pentapartitioned or heptapartitioned single-valued neutrosophic sets.

Funding

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Conflicts of interest

The authors declare that there is no conflict of interest.

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